DISCLAIMER –

I have prepared this document by collecting information primarily from ICFAI material for my preparation. I do not claim any copyright on this document. I do not vouch for its correctness or usefulness. Please use at your own risk.

If you need any help, you may contact me at www.hanumant.com

This document has 3 parts –

1. **Tips for Objective Questions**
2. **Big Questions – These are the numerical questions that are to be solved step by step in the exam. I have gone through previous years question papers and have collected all such questions that, taken together, cover the whole syllabus. I have found this to be of great use because it saved me from solving the workbook. I haven't found any question out of this set in past two examinations (the ones conducted after I prepared these notes.)**
3. **Theory Questions – These are the theoretical question from past years question papers. These questions are important ones. Many of these are often repeated. However, ICFAI has a tendency to ask one or two entirely new questions in each question paper that are not answered anywhere in the book or workbook or past question papers.**

Tips for Objective Questions

Bond at discount: YTM > current yield > coupon yield
Bond at premium: coupon yield > current yield > YTM
Bond at par: YTM = current yield = coupon yield.

I. The coupon of bond and convexity are inversely related.
II. The maturity of a bond and convexity are directly related.
III. The yield of a bond and convexity are inversely related.

Various forms of internal credit enhancements are
- Reserve funds
- Overcollateralization
- Senior/subordinated Structures

Bond Insurance is external credit enhancement.

YTM is measured by comparing the present value of coupon payments and redemption of the bond by the issuer at a **contracted value** and **not** the value at which it is sold to the prevailing market price.

Imputed Interest

Purchase price $= \frac{1000}{1.08^{20}} = 214.54$

$\therefore$ Imputed interest income in the second year

$= 214.54 \times 1.08 \times .08 = 18.54$

Imputed interest income of the last year

$= 214.54 \times 1.08^{19} \times 0.08 = 74.07$

The holding period at which the terminal value of the bond will remain same irrespective of the change in the reinvestment rate can be described as duration.

Calculation for duration is as follows: (Assume Reinvestment rate of 12.5%.)

$$D = \frac{r_c}{r_d} \times PVIFA_{(r_d, n)} \times (1 + r_d) + \left[1 - \frac{r_c}{r_d}\right] n$$

$$= \frac{0.125}{0.125} \times PVIFA_{(12.5\%, 5)} \times (1.125) + \left[1 - \frac{0.125}{0.125}\right] \times 5$$

= 4.

Pure expectations theory - $(1 + r_{0,2})^2 = (1 + r_{0,1})(1+f_{1,2})$

Liquidity Premium Theory - According to liquidity premium theory investors are not indifferent to risk and they charge higher rates than the expected future rates, if the maturity of the instrument increases.

Modified Duration

Active and Passive Investment strategies
(a) Yield spread strategies
(b) Cash flow matching strategies
(c) Constant duration strategies
(d) Yield curve strategies
(e) Return Enhancement strategies.

For a given difference between YTM and coupon rate of the bonds, the longer the term to maturity, the greater will be the change in price with change in YTM.

The percentage price change increases at a diminishing rate as the bond's maturity time increases.

The increase in the price of a bond associated with the changes in the interest rates will be at a diminishing rate as the term to maturity increases.

For a given change in a bond's YTM, the percentage price change will be **higher** for **low coupon** bonds than for high coupon bonds

Convexity is used for capturing the changes in prices for both large and small shanges in the required yield.

Relationship between the convexity, coupon, maturity and yield of a bond:

i. the coupon of a bond and convexity are inversely related.
ii. The maturity of a bond and convexity are directly related.
iii. The yield of a bond and convexity are inversely related.

The determinants of the duration and convexity for option-free bonds are similar and the direction of impact is also similar. Therefore, higher the duration of the bond, higher the convexity.

Secured debentures normally carry a fixed or floating charge on the immovable assets of the company by way of an equitable mortgage

Unregistered debentures are freely transferable and can be transferred by a simple endorsement, while the registered debentures can be transferred only by executing a transfer deed and filing a copy of it with the company.
Convertible zero-coupon bond is redeemed by allocation of ordinary shares
Debenture Redemption Reserve has to be created by the company out of its profits to the extent of 50% of the amount of debentures to be redeemed before the date of redemption
In case of debentures with call option, the call price is maximum at the start of the effective call option period and declines step-wise towards the face value as the call date approaches the maturity date.

According to Liquidity Preference Theory, spenders keep a proportion of their assets as cash balances for maintaining liquidity.
According to this Pure Expectations Theory, the current term structure of interest rates is determined by the consensus forecast of future interest rates.
According to the loanable funds theory, interest rates in different sectors in the economy can be predicted as the theory focuses on the demand and supply aspects of the funds in an economy with different sectors like the household sector, the business sector and the government sector.
According to the Liquidity Premium Theory, the investors are not indifferent to risk and they charge higher rates than the expected future rates, if the maturity of the instrument increases.

According to the Preferred Habitat Theory, it is not necessary that the liquidity premium should increase at a uniform rate with maturity.

Implied Price
The price computed by a model which considers a comparable benchmark, volatility, and spread adjustment. It is used in the absence of a current market price.

CIR Model cannot be implemented by regressing cross-sections of bond returns and their durations. This is the disadvantage in this model.
Principal Components Models can be used to analyze the returns of zero-coupon bonds of varying maturities to extract a set of characteristic yield curve shifts, which can be defined at each maturity.
Spot Rate Models can be used to identify factors with the durations of zero-coupon bonds at several points on the yield curve.
Functional Models assume that zero-coupon yield changes are defined continuously in maturity.

The major drawback of effective duration measure is that it is calculated based on the assumption of parallel shifts in the yield curve. To overcome this an alternative measure used for different types of yield curve shifts is "Key Rate Duration" popularly referred to as KRD.

KRD can identify the price sensitivity of an option embedded bond to each segment of the spot yield curve.
The sum of KRDs will be equal to the effective duration.
KRD can capture the influence of multiple market factors on the yield curve movement.
KRD can be used to replicate a portfolio of a bond with embedded options created with zero-coupon bonds.
~~The major drawback of KRD is that it is calculated based on the assumption of parallel shifts in the yield curve.~~

Duration and Modified Duration
Duration is a measure of the average (cash-weighted) term-to-maturity of a bond. The are two types of duration, Macaulay duration and modified duration. Macaulay duration is useful in immunization, where a portfolio of bonds is constructed to fund a known liability. Modified duration is an extension of Macaulay duration and is a useful measure of the sensitivity of a bond's price (the present value of it's cash flows) to interest rate movements.

$$\text{Modified Duration} = \left[\frac{\text{Macaulay Duration}}{\left(1+\frac{\text{YTM}}{n}\right)}\right]$$

Effective Duration
The modified duration formula discussed above assumes that the expected cash flows will remain constant, even if prevailing interest rates change; this is also the case for option-free fixed-income securities. On the other hand, cash flows from securities with embedded options or redemption features will change when interest rates change. For calculating the duration of these types of bonds, effective duration is the most appropriate.

Effective duration requires the use of binomial trees to calculate the option-adjusted spread (OAS).

Key-Rate Duration
The final duration calculation to learn is key-rate duration, which calculates the spot durations of each of the 11 "key" maturities along a spot rate curve. These 11 key maturities are at the three-month and one, two, three, five, seven, 10, 15, 20, 25, and 30-year portions of the curve.

In essence, key-rate duration, while holding the yield for all other maturities constant, allows the duration of a portfolio to be calculated for a one-basis-point change in interest rates. The key-rate method is most often used for portfolios such as the bond ladder, which consists of fixed-income securities with differing maturities.

Option Adjusted Spread

$$= \frac{Price_{NCB}}{Price_{CB}} \times Dur_{NCB}(1 - Delta)$$

$$= (112/105.45) \times 4.5 \times (1 - 0.672) = 1.567$$

$$\text{option-adjusted spread} = \text{spread} - \text{spread due to optionality}$$

$$\text{bond's yield} = \text{spread} + \text{benchmark yield}$$

$$= \text{spread due to optionality} + \text{option-adjusted spread} + \text{benchmark yield}$$

The higher the expected interest rate volatility, the lower the OAS. Similarly the lower the expected interest rate volatility, the higher the OAS.
It is a measure of the yield spread, which can be used to convert differences between the values and the prices.
It is basically used as a tool to reconcile value with market price.
The cash flows of the callable bond are adjusted to reflect the embedded option; the resulting spread is called option adjusted spread.

For perfect immunization term-structure should not be upward sloping but it should be flat.
The investment is made in default free bondBuy-and-hold strategy is adopted
There is only a time change in interest rate during the investment horizon.
The duration of a bond is matched with the investment horizon.

Big Questions

Q.

Two years ago Indian Photofilms Ltd. had issued bonds which have a book value of Rs.5 crores. These bonds carry an interest rate of 12% per annum payable semi-annually and are redeemable in two equal annual installments, each amounting to 50% of the par value of the bonds, payable at the end of the last two years of the life of the bonds. These bonds have a maturity period of 7 years. The yield to maturity on these bonds prevailing in the market is 10%.

You are **required** to find out the aggregate market value of the bonds at present.

(12 marks)

Answer

The value of the bond can be found out as follows:

$$V = \sum_{t=1}^{2n} \frac{I/2}{(1+k_d/2)^t} + \frac{F}{(1+k_d/2)^{2n}}$$

Since the redemption value of the bond is payable in two equal installments, the first term as well as the second term on the RHS have to be modified.

Given, F = book value = Rs.500,00,000

I = 500,00,000 (0.12) = Rs.60,00,000

I/2 = Rs.30,00,000

k_d = 10% = 0.10

$k_d/2$ = 5% = 0.05

n = 7 – 2 = 5

(Since the bonds were issued two years ago, two years have to be deducted from the maturity period of seven years).

2n = 2×5 = 10

V = 30,00,000 $PVIFA_{(5\%, 8)}$ + 2,50,00,000 $PVIF_{(5\%, 8)}$ + 15,00,000 $PVIF_{(5\%, 9)}$ + 15,00,000 $PVIF_{(5\%, 10)}$ + 250,00,000 $PVIF_{(5\%, 10)}$

$$= 30,00,000\ PVIFA(5\%, 8) + \frac{15,00,000}{(1.05)^9} + \frac{15,00,000}{(1.05)^{10}} + \frac{250,00,000}{(1.05)^8} + \frac{250,00,000}{(1.05)^{10}}$$

= 30,00,000 (6.463) + 15,00,000 [0.645 + 0.614] + 250,00,000 [0.677 + 0.614]

= Rs.53,552,500

A bond issued by Greaves Metal Ltd. is selling presently at a face value of Rs.100 and pays coupon rate at the rate of 13% p.a. in arrears which will be redeemed at Rs.113 after five years. The 'n' years spot rate of interest, yn is given by yn(%) = $8.5 + \frac{n}{6}$ where, n = 1, 2 3, 4 and 5. The term structure of interest rates is flat and pure expectation theory holds good.

You are **required** to calculate

a. The implied one year forward rates applicable at time t = 3 and t = 4 .

b. The value of the bond at time 0.

c. The duration of the above bond.

d. Change in bond price for 50 basis point increase in interest rates.

(3 + 3 + 3 + 3 =12 marks)

Answer

a. We can find the forward rates by f_3 and f_4 using spot rate y_1, y_2, y_3, y_4, and y_5:

$(1+y_4)^4 = (1+y_3)^3(1+f_3)$ and

$(1+y_5)^5 = (1+y_4)^4(1+f_4)$

For, f_3

$(1.0917)^4 = (1.09)^3(1+f_3)$

$f_3 = 9.68\%$

For, f_4

$(1.0933)^5 = (1.0917)^4(1+f_4)$

$f_4 = 9.97\%$

b. The value of the bond at time = 0

$13(PVIF_{8.67\%,1} + PVIF_{8.83\%,2} + PVIF_{9..0\%,3} + PVIF_{9.17\%,4} + PVIF_{9..33\%,5}) + 113 \times PVIF_{9.33\%,5}$

$= 13(0.920 + 0.844 + 0.77 + 0.704 + 0.640) + 113 \times 0.640$

$= 122.76$

c. $100 = 13\ PVIFA_{(r\ 5)} + 113\ PVIF_{(r\ 5)}$

r = 15%

Duration

$$\frac{r_c}{r_d} \times PVIFA_{(r_d,n)} \times (1+r_d) + \left(1 - \frac{r_c}{r_d}\right) \times n$$

$$= \frac{0.13}{0.15} \times PVIFA_{(15\%,5)} \times (1.15) + \left(1 - \frac{0.13}{0.15}\right) \times 5$$

$= 3.34 + 0.67$

$= 4.01$ years

$$D_{mod} = \frac{D}{1+\frac{r_d}{f}} = \frac{4.01}{1.15} = 3.49 \text{ years}$$

$$\frac{\Delta P_o}{P_o} \times 100 = -D_{mod} \times \frac{\Delta BP}{100}$$

= - 3.49 years $\times$ 50/100 = -1.745%

i.e.. 1.745% fall in bond price.

Therefore new price = 100 (1 - 0.01745) = Rs. 98.26

Mr. Mukherjee is holding two bonds A and B which pay an annual coupon of 6% and 8% and their terms to maturity are 4 years and 5 years, respectively. The face value and maturity value of the bonds is Rs.1000. Spot rates prevailing in the market as indicated by the yield curve are:

Maturity (Years)	Spot rates
1	4.00%
2	5.00%
3	5.60%
4	6.10%
5	6.75%

You are **required** to calculate

a. The expected change in the prices of bonds A and B using the duration concept, for a 0.40% change in yield to maturity.

b. The one year holding period return on the bonds assuming that spot rates will rise in twelve month's time by 0.15%, across the maturity spectrum.

(8 + 2 = 10 marks) < Answer >

Answer

(This answer has mistakes in calculating duration of A)

Price of the bond B

$$P = \frac{80}{(1.04)^1} + \frac{80}{(1.05)^2} + \frac{80}{(1.056)^3} + \frac{80}{(1.061)^4} + \frac{1080}{(1.0675)^5}$$

= 76.923 + 72.562 +67.936 + 63.129 + 779.08

= 1059.63

Yield to maturity of the bond B

$$1059.6 = \frac{80}{(1+k)^1} + \frac{80}{(1+k)^2} + \frac{80}{(1+k)^3} + \frac{80}{(1+k)^4} + \frac{1080}{(1+k)^5}$$

K = 6.56%

Duration of the Bond B

Year	C.F	Present value of cash flow at (6.56%)	Year x PVCF
1	80	75.075	75.0751
2	80	70.453	140.907
3	80	66.116	198.348
4	80	62.046	248.184
5	1080	786.05	3930.27
		1059.7	4592.79

$$\text{Duration} = \frac{4592.79}{1059.7}$$

= 4.334 years

$$\text{Modified duration} = \frac{4.334}{1+0.0656}$$

= 4.067 years

Change in the price of the bond = –4.06727 x0.40

= – 1.6268%

Therefore, price of the bond B will change by 1.6268%.

b. Price of bonds after 1 yr

$$\frac{60}{(1+k)^1} + \frac{60}{(1+K)^2} + \frac{1060}{(1+K)^3}$$

57.609 + 54.267 +896.32 = Rs.1008.2

$$\text{One year holding period return on bond A} = \frac{1008.2 - 1000 + 60}{1000} = 6.82\%$$

Bond B

$$\frac{80}{(1+k)^1}+\frac{80}{(1+k)^2}+\frac{80}{(1+k)^3}+\frac{1080}{(1+k)^4}$$

= 76.812 + 72.355 + 67.647 + 847.44= 1064.3

One year holding period return = $\frac{1064.3-1059.7+80}{1059.7}$ =7.98%

Consider a 9% bond (face value Rs.1000) redeemable after 5 years at a premium of 6%. Currently the bond is available in the market at a price of Rs.1124.80.

You are **required** to:

a. Calculate the interest on interest at reinvestment rate of 7% and 10% for various possible holding period.
b. Calculate expected market price for various possible holding period at the reinvestment rate of 7% and 10% and the capital gain that would arise, if the bond were sold at that price.
c. Calculate the total return from bond, classifying the each income on bond, for various possible holding period at the reinvestment rate of 7% and 10%.
d. Interpret the effect of reinvestment rate on the total return.

Answer

2. a.

Reinvestment rate	1	2	3	4	5
7%	$90 \times FVIFA_{(7\%, 1)} - (90 \times 1)$	$90 \times FVIFA_{(7\%, 2)} - (90 \times 2)$	$90 \times FVIFA_{(7\%,3)} - (90 \times 3)$	$90 \times FVIFA_{(7\%, 4)} - (90 \times 4)$	$90 \times FVIFA_{(7\%, 5)} - (90 \times 5)$
	= 0	=6.30	=19.34	=39.59	=67.56
10%	$90 \times FVIFA_{(10\%, 1)} - (90 \times 1)$	$90 \times FVIFA_{(10\%, 2)} - (90 \times 1)$	$90 \times FVIFA_{(10\% 3)} - (90 \times 1)$	$90 \times FVIFA_{(10\%, 4)} - (90 \times 1)$	$90 \times FVIFA_{(10\%, 5)} - (90 \times 1)$
	=0	=9.00	=27.90	=57.69	=99.46

b. Market price and capital gains @ 7%

At reinvestment rate of 7%	Holding period (Years)				
	1	2	3	4	5
1. Purchase price (Rs.)	1124.80	1124.80	1124.80	1124.80	1124.80
2. Market price at the end of the holding period (Rs.)	$90 \times PVIFA_{(7\%, 4)} + 1060 \times PVIF_{(7\%, 4)}$	$90 \times PVIFA_{(7\%, 3)} + 1060 \times PVIF_{(7\%,3)}$	$90 \times PVIFA_{(7\%, 2)} + 1060 \times PVIF_{(7\%, 2)}$	$90 \times PVIFA_{(7\%, 1)} + 1060 \times PVIF_{(7\%, 1)}$	$90 \times 0 + 1060$
	= 1113.52	= 1101.47	= 1088.52	= 1074.79	= 1060
3. Capital gain (2) – (1) (Rs.)	-11.28	-23.33	-36.28	-50.01	-64.8

Market price and capital gains @ 10%

At reinvestment rate of 10%	Holding period (Years)				
	1	2	3	4	5
1. Purchase price (Rs.)	1124.80	1124.80	1124.80	1124.80	1124.80
2. Market	$90 \times PVIFA_{(10\%, 4)}$	$90 \times PVIFA_{(10\%, 3)}$	$90 \times PVIFA_{(10\%, 2)}$	$90 \times PVIFA_{(10\%, 1)}$	90×0

price at the end of the holding period (Rs.)	+ 1060 × PVIF (10%, 4)	+ 1060 × PVIF (10%,3)	+ 1060 × PVIF (10%, 2)	+ 1060 × PVIF (10%, 1)	+ 1060
	= 1009.27	= 1020.20	= 1032.18	= 1045.47	= 1060
3. Capital gain (2) – (1) (Rs.)	–115.53	–104.60	–92.62	–79.34	–64.80

c. Total Return

Reinvestment rate		Holding Period (Years)				
		1	2	3	4	5
7% (YTM)	Coupon income	90	180	270	360	450
	Interest on interest	0	6.30	19.34	39.59	67.56
	Capital gains	-11.28	-23.33	-36.28	-50.01	-64.8
	Total return	78.72	162.97	253.06	349.58	452.76
10% (YTM)	Coupon income	90	180	270	360	450
	Interest on interest	0	9.00	27.90	57.69	99.46
	Capital gains	–115.53	–104.60	–92.62	–79.34	–64.80
	Total return	–25.53	84.4	205.28	338.35	484.66

d. We can see the combined effect of varying the holding period and, and the reinvestment rate, on the total return. It can be observed that two opposing forces work on the return. A fall in interest rates reduces interest on interest, while increasing the capital gains or decreasing the capital loss. This is due to the inverse relationship between YTM and the market prices.

Mr. Nandagopal Reddy is holding two bonds A and B with an annual coupon of 7% and 8.5% and their terms to maturity are 4 years and 6 years respectively. The face value and maturity value of the bonds is Rs.100. Spot rates prevailing in the market as indicated by the yield curve are:

Maturity (in years)	Spot rates (in %)
1	5.62
2	6.05
3	6.30
4	6.43
5	6.58
6	6.72

You are **required** to:

a. Calculate the expected change in the prices of bonds A and B for a 0.75% change in yield to maturity, using the convexity concept.

b. Calculate the one year holding period return on the bonds assuming that spot rates will fall in twelve month’s time by 0.25% across the maturity spectrum.

(10 + 2 = 12 marks)

Answer

$$P = \frac{7}{(1.0562)} + \frac{7}{(1.0605)^2} + \frac{7}{(1.0630)^3} + \frac{107}{(1.0643)^4}$$

P = 6.63 + 6.22 + 5.83 + 83.40
= Rs.102.08

YTM of the bond A

$$102.08 = \frac{7}{(1+k)^1} + \frac{7}{(1+k)^2} + \frac{7}{(1+k)^3} + \frac{107}{(1+k)^4}$$

At K = 7%
LHS = 100
At K = 6%
LHS =103.47
By trial and error method, YTM is 6.40%.

Year	CF_t	PV @ 6.4%	PV (CF)	$t^2 + t$	(4) × (5)
(1)	(2)	(3)	(4)	(5)	(6)
1	7	0.9398	6.58	2	13.16
2	7	0.8833	6.18	6	37.08
3	7	0.8302	5.81	12	69.72
4	7	0.7802	5.46	20	109.20
4	100	0.7802	78.02	20	1560.40
Total			102.05		1789.56

$$\text{Convexity} = \frac{1}{P(1+i)^2}\left[\sum_{t=1}^{n}\frac{CF_t}{(1+i)^t}(t^2+t)\right]$$

$$\frac{1}{(1+i)^2} = \frac{1}{(1.064)^2} = 0.8833$$

1789.56 × 0.8833 = 1580.72

Therefore convexity = $\frac{1580.72}{102.05}$ = 15.49.

Price change due to convexity = 1/2 × Price × Convexity × (Change in Yield)^2

Therefore Price change for 0.75% change in yield
= ½ × 102.05 × 15.49 × (0.0075)^2
=4.45%.

Price of the bond B

$$P = \frac{8.50}{(1.0562)^1} + \frac{8.50}{(1.0605)^2} + \frac{8.50}{(1.0630)^3} + \frac{8.50}{(1.0643)^4} + \frac{8.50}{(1.0658)^5} + \frac{108.50}{(1.0672)^6}$$

= 8.05 + 7.56+ 7.08+ 6.62+ 6.18+ 73.44= Rs.108.93

Yield to maturity of the bond B

$$108.93 = \frac{8.50}{(1+K)^1} + \frac{8.50}{(1+K)^2} + \frac{8.50}{(1+K)^3} + \frac{8.50}{(1+K)^4} + \frac{8.50}{(1+K)^5} + \frac{108.50}{(1+K)^6}$$

at = K = 6%

L.H.S. = 112.30

at = K = 7%

L.H.S = 107.18

YTM = 6% + $\frac{(112.30-108.93)}{(112.30-107.18)}$

= 6.66%.

Year	CF_t	PV @ 6.66%	PV (CF)	$t^2 + t$	(4) (5)
(1)	(2)	(3)	(4)	(5)	(6)
1	8.50	0.9376	7.97	2	15.94
2	8.50	0.8790	7.47	6	44.82
3	8.50	0.8241	7.00	12	84
4	8.50	0.7747	6.57	20	131.4
5	8.50	0.7244	6.16	30	184.5
6	8.50	0.6792	5.77	42	242.34
6	100	0.6792	67.92	42	2852.64
Total			108.86		3555.94

Convexity = $\frac{1}{P(1+i)^2}\left[\sum_{t=1}^{n}\frac{CF_t}{(1+i)^t}(t^2+t)\right]$

$\frac{1}{(1+i)^2} = \frac{1}{(1.0667)^2} = 0.8789$

3555.94 × 0.8789 = 3125.32

Therefore convexity = =28.71.

Price change due to convexity = 1/2 × Price × Convexity × (Change in Yield)^2

Therefore Price change for 0.75% change in yield

= ½ × 108.86 × 28.71 × $(0.0075)^2$

=8.79%.

b.

Price of the bonds after one year

Bond A

$\frac{7}{(1.0537)^1}+\frac{7}{(1.0580)^2}+\frac{107}{(1.0605)^3}$

= 6.64 + 6.25 + 89.71 = 102.60

One year holding period return on bond A = $\frac{102.60-102.05+7}{102.05}$ = 7.40%.

Bond B

$\frac{8.50}{(1.0537)}+\frac{8.50}{(1.0580)^2}+\frac{8.50}{(1.0605)^3}+\frac{8.50}{(1.0618)^4}+\frac{108.50}{(1.0633)^5}$

= 8.07+7.59+7.13+6.69+79.83=109.31

One year holding period return = $\frac{109.31-108.86+8.50}{108.86}$ = 8.22%.

Mr. Prashant will have an annual cash inflow of Rs.4,00,000 for four years from the end of three years from now. Mr. Prashant wants to invest these receipts in the 10% coupon-bearing bond of the face value of Rs.1,000, maturing after 5 years, redeemable at par value and is currently traded at Rs.970.

Mr. Prashant has Rs.20,00,000, which he is planning to invest in zero-coupon bonds of 2 and 10 years respectively. Mr. Prashant wants the modified duration for these two zero-coupon bonds to be same as for coupon bearing bond above. The coupon bearing bond of 2 and 10 years maturity have interest rate of 6.5% and 9.00% respectively.

You are **required** to calculate the proportion of fund to be invested in the 10-year zero-coupon bonds and also the face value to be purchased by Mr. Prashant for the same. (Assume that the zero-coupon bonds can be purchased for the theoretical price and the face value is Rs.100).

Answer

Duration of 10% coupon bearing bond:

YTM is 'r_d' in the following:

100 $PVIFA_{rd\%, 5}$ + 1000 $PVIF_{rd\%,5}$ = Rs.970

If r_d is 10%, LHS = Rs.1000

If r_d is 12%, LHS = Rs.927.88

Therefore, r_d = 10.83%

r_c = current yield = $\frac{100}{970}$ = 10.31%

$$\text{Duration} = \frac{r_c}{r_d} PVIFA_{rd,n}(1 + r_d) + \left[1 - \frac{r_c}{r_d}\right] n$$

$$= \frac{0.1031}{0.1083} PVIFA_{(10.83\%,5)}(1.1083) + \left[1 - \frac{0.1031}{0.1083}\right] \times 5$$

= (0.9520) (3.7117) (1.1083) + 0.2401

= 4.16 years (approx.)

Modified duration for the bond = 4.16 / 1.1083 = 3.7535 years

2-year zero-coupon bond modified duration: 2÷(1+0.065)=1.8779

10-year zero-coupon bond modified duration: 10÷(1+0.09)=9.1743

Let 'w' be the weight of 10-year zero-coupon bonds

((1-w) · 1.8779) + (w · 9.1743) = 3.7535

(9.1743 – 1.8779) · w = 3.7535 – 1.8779

7.2964 · w = 1.8756

w = 1.8756 / 7.2964 = 0.2571 = 25.71%

Theoritical value of zero-coupon bonds can be found as follows:

100 = P_0 $(1.09)^{10}$

P_{0} = Rs.42.24

∴Face value of 10-year zero-coupon bonds

$$F = \frac{20,00,000}{42.24} \times 0.2571 \times 100 = \text{Rs.}12,17,329.55$$

Consider the following zero-coupon curve:

Maturity (years)	Zero-coupon rate (%)
1	5.00
2	5.50
3	5.75
4	5.90
5	6.00

An investor is interested in purchasing a 5-years bond with a face value of Rs.100, which delivers coupons in the following manner:

Years	Coupon (%)
1 - 2	8.0
3 - 4	8.5
5	9.0

You are **required** to

a. Compute the price and YTM of this bond at the present term structure and when the zero-coupon curve increases instantaneously and uniformly by 0.5%. Indicate the impact of this rate rise for the bondholder in both absolute and relative terms.

b. Compute the investor's annual rate of return considering that the zero-coupon curve remains stable over time and the investor holds the bond until maturity and reinvests the coupons at the zero-coupon rates.

c. Compute the rate of return earned by the investor, if he purchases a one-year, two-year and a three-year zero coupon bond and sell all bonds after a year. The investor expects the yield curve to shift to left by 25 basis point across the zero coupon bonds after one year.

(3 + 2 + 3 = 8 marks)

Answer

a. The price P of the bond is equal to the sum of its discounted cash flows and given by the following formula

$$P = \frac{8}{(1+5\%)} + \frac{8}{(1+5.5\%)^2} + \frac{8.5}{(1+5.75\%)^3} + \frac{8.5}{(1+5.9\%)^4} + \frac{109}{(1+6\%)^5}$$

$$= Rs.110.20$$

YTM on this bond:

$$110.20 = \frac{8}{(1+r)} + \frac{8}{(1+r)^2} + \frac{8.5}{(1+r)^3} + \frac{8.5}{(1+r)^4} + \frac{109}{(1+r)^5}$$

By trial and error, r = 5.95%

If the yield curve rises by 50 basis points, i.e., by 0.5%, price of the bond:

$$P = \frac{8}{(1+5.5\%)} + \frac{8}{(1+6\%)^2} + \frac{8.5}{(1+6.25\%)^3} + \frac{8.5}{(1+6.4\%)^4} + \frac{109}{(1+6.5\%)^5}$$

$$= Rs.107.98$$

YTM on this bond:

$$107.98 = \frac{8}{(1+r)} + \frac{8}{(1+r)^2} + \frac{8.5}{(1+r)^3} + \frac{8.5}{(1+r)^4} + \frac{109}{(1+r)^5}$$

By trial and error, r = 6.28%.

Impact of this rate rise would cause a loss to the bond holder in both absoute and relative terms:

Absolute Loss = Rs.107.98 – Rs.110.20 =-Rs.2.22

Relative Loss = $\frac{-2.22}{110.20}$ x 100 = -2.014%

b. Annual Rate of Return for the bond holder, if he reinvests the coupons in the market:

- After one year, he receives Rs.8.00, which he can reinvest for 4 years at the 4-year zero-coupon rate to obtain on the maturity date of the bond.

  8 x $(1 + 5.9\%)^4$ = Rs.10.06

- After two years, he receives Rs.8.00, which he can reinvest for 3 years at the 3-year zero-coupon rate to obtain on the maturity date of the bond.

  8 x $(1 + 5.75\%)^3$ = Rs.9.46

- After three years, he receives Rs.8.50, which he can reinvest for 2 years at the 2-year zero-coupon rate to obtain on the maturity date of the bond.

  8.5 x $(1 + 5.5\%)^2$ = Rs.9.46

- After four years, he receives Rs.8.50, which he can reinvest for 1 year at the 1-year zero-coupon rate to obtain on the maturity date of the bond.

  8.5 x (1 + 5%) = Rs.8.925

- After five years, he receives the final cash flow equal to Rs.109.

The bond holder finally receives Rs.146.905 five years later, i.e.,

10.06 + 9.46 + 9.46 + 8.925 + 109 = Rs.146.905.

Annual rate of return = $\left(\frac{146.905}{110.20}\right)^{1/5} - 1 = 5.918\%$

c. The 1-year zero coupon is purchased at (Rs.100/1.05) and will be redeemed at par

The 2-year zero coupon bond has been purchased at (Rs.100/1.055^2) and will be sold at (Rs.100/1.0525)

The 3-year zero coupon bond has been purchased at (Rs.100/1.0575^3) and will be sold at (Rs.100/1.0575^2)

Now,

Return on 1-year zero coupon bond = $\frac{100 - 95.23}{95.23}$ = 5.00%

Return on 2-year zero coupon bond = $\frac{95.012 - 89.845}{89.845}$ = 5.75%

Return on 3-year zero coupon bond = $\frac{89.421 - 84.559}{84.559}$ = 5.75%

An analyst is considering an investment in the structured products. An investment in a 5-year bond with 50 warrants per Rs.10,000 is proposed. The coupon rate for the bond is 9.5% and the bond is trading at par value of Rs.100. Five warrants give right to buy 1 stock of the company at Rs.200. Currently the stock is trading at Rs.320.

You are **required** to

a. Calculate the price of the bond without warrants, if the market indicates a redemption yield for bonds of the same quality and maturity of 12%.

b. Calculate the implicit price of one warrant. Give the reason for any difference from the intrinsic value.

c. Discuss how price volatility of the share is attached with the pricing of the warrant.

Answer

a. Price = $\frac{9.5}{1.12} + \frac{9.5}{1.12^2} + \frac{9.5}{1.12^3} + \frac{9.5}{1.12^4} + \frac{109.5}{1.12^5}$ = 90.98 ≅ Rs.91

b. Because the bond is trading at par Rs.100, the difference in price between Rs.100 and Rs.91 (calculated above in a) must be due to the warrants. With 50 warrants per Rs.10000 nominal value, the implicit cost per warrant is 10000*(100%-91%)/50 = Rs.18. This means that 1 call on the underlying stock with strike 200 costs Rs.90 (5 warrants are needed). Since the stock is trading at 320, the intrinsic value of the call is 120. Therefore the warrants seem to be undervalued.

The reasons for the undervaluation:

1. The yield of 12% is wrong and should be higher.
2. The warrants may be of European style and in between the company may have paid large dividend and therefore the price of the warrant already discounts.

c. As the price volatility of the underlying share increases, the premium of the warrant also increases. Because the investors are ready to pay extra for such warrants due to speculative reasons, their expectancy of increase in prices of underlying stock also increases, which lures them to purchase such warrants.

Mr. Ravi Jain estimates that there will be semi-annual cash outflows of Rs.25,000 for a period of three and half years, the first payment of which will commence from the end of two and half years from now. There is a cash outflow of Rs.1,20,000 at the end of sixth year from now. The cost of debt for similar debt in the market is 13%. Mr. Ravi Jain is considering immunizing the cash flows by investing in the two bonds i.e., Lakshmi Metals Ltd. (LML) bond and Government of Gujarat (GOG) bond. LML bond is a 12% semi-annual coupon bearing bond with a face value of Rs.100, maturing after 6 years, redeemable at a premium of 5% and is currently trading at Rs.106.94. Whereas GOG bond is a Zero coupon bond with a face value Rs.1000 maturing after 7 years and is currently traded at Rs.630.17.

As an analyst, you are **required** to determine the proportion of funds to be invested by Mr. Ravi in the LML and GOG bonds such that his payments are immunized.

Answer

Duration of outflows:

Therefore duration can be calculated for the following liability as under:

YTM = 13% (6.5% for semi-annual)

No.	Cash outflow	PVIF @ 6.5%	
(1)	(2)	(3)	(1)*(3)
5	25000	18247.02	91235.10
6	25000	17133.35	102800.12
7	25000	16087.66	112613.59
8	25000	15105.78	120846.24
9	25000	14183.83	127654.48
10	25000	13318.15	133181.51
11	25000	12505.31	137558.37
12	120000	56361.94	676343.31
		162943.04	**1502232.71**

Therefore, Duration = 1502232.71/162943.04 = 9.22 i.e. 4.61 years

Duration of Inflows:

LML bond:

Firstly, we have to calculate the YTM for the bond

106.94 = 6 PVIFA(12,i) + 105 PVIF(12,i)

By trial and error, we get i = 5.5%

Period	Cash outflow	PVIF @ 5.5%	
(1)	(2)	(3)	(1)*(3)
1	6	5.69	5.69
2	6	5.39	10.78
3	6	5.11	15.33
4	6	4.84	19.37
5	6	4.59	22.95
6	6	4.35	26.11
7	6	4.12	28.87
8	6	3.91	31.28
9	6	3.71	33.35
10	6	3.51	35.13
11	6	3.33	36.62
12	111	58.38	700.61
		106.94	**966.09**

Therefore, Duration = 966.09/106.94 = 9.03 i.e. 4.52 years

GOG bond:

Duration for GOG bond will be same as its maturity as it is zero coupon bond. Therefore, duration of GOG is 6 years.

Now, to immunize the payments, duration of investment = duration of liabilities.

Let proportion of funds to be invested in LML bond be x.

Then, the duration of investment = x × 4.52 + (1 – x) 6

As duration of investment = Duration of liabilities

4.52x – 6x + 6 = 4.61

Therefore, x = 94%

Proportion of funds in LML bond = 94% and GOG bond = 6%

Mr. Biswajit expects that there will be annual cash outflows of Rs.1,20,000 for five years, starting from end of the second year from now. Mr. Biswajit wants to immunize the liability and is actively considering the following two bonds:

Bond A : A zero-coupon bond with a face value Rs.1,000, maturing after 6 years and is currently trading at Rs.506.63.

Bond B : 9% coupon bearing bond with a face value of Rs.1,000, maturing after 4 years, redeemable at par value and is currently trading at Rs.908.86.

The yield curve is expected to be stable in the near future.

You are **required** to

a. Determine the proportion of funds to be invested in the bonds A and B so that the liability of Mr. Biswajit is immunized.
b. If the interest declines by 1%, estimate the interest rate elasticity and interest rate risk associated with the bonds.
c. State whether the proportion of investment recommended in (a) above offer perfect immunization and also give conditions to be fulfilled for perfect immunization.

Answer

YTM to be found out by solving any of the bond by trial and error method. Therefore YTM=12%

a. Duration of outflows:

Year	Cash outflows Rs.	PVIF @ 12%	PV. Rs.	Proportion of (4) in total PV	(1) x (5)
(1)	(2)	(3)	(4)	(5)	(6)
2	1,20,000	0.7972	95,664	0.2477	0.4954
3	1,20,000	0.7118	85,416	0.2212	0.6635
4	1,20,000	0.6355	76,260	0.1975	0.7898
5	1,20,000	0.5674	68,088	0.1763	0.8815
6	1,20,000	0.5066	60,792	0.1574	0.9444
			3,86,220		3.7746

- Duration of bond A:

  As it is zero coupon bond, its duration is equal to years to maturity = 6 years.

- Duration of Bond B:

YTM is 'r_d' in the following:

$$90 PVIFA_{rd,4} + 1000 PVIF_{rd,4} = 908.857$$

$$\therefore r_d = 12\%$$

r_c = Current yield $= \dfrac{90}{908.86} = 0.0990$

Duration $= \dfrac{r_c}{r_d} PVIFA_{rd,n}(1+r_d) + [1 - \dfrac{r_c}{r_d}]n$

$$= \frac{0.0990}{0.12} PVIFA_{(12,4)}(1.12) + [1 - \frac{0.0990}{0.12}] \times 4$$

= 3.5065 years

To immunize the payments, duration of investment = Duration of liabilities

Let proportion of funds to be invested in bond A be X. Then, the distribution of investment

= 6(X) + 3.5065(1- X)

As duration of investment = Duration of liabilities

6X + 3.5065 – 3.5065X = 3.7746

X = 10.75% and 1 – X = 89.25%

Proportion of fund in Bond A = 10.75% and Bond B = 89.25%.

b. Bond A:

Duration = 6 years

IE = $D \times \frac{K_d}{1+K_d}$

IE = $6 \times \frac{0.12}{1.12} = -0.6429$

Interest rate risk = $\left\{\frac{\Delta p_0}{p_0}\right\} = IE_{it}\left\{\frac{\Delta YTM}{YTM}\right\} = -0.6429 \times \frac{0.01}{0.12} = 3.358\%$

Bond B:

Duration = 3.5065 years

IE = $3.5065 \times \frac{0.12}{1.12} = -0.3757$

Interest rate risk = $= 0.3757 \times \frac{0.01}{0.12} = 3.131\%$

b. Proportion of investment recommended in (a) will be giving perfect immunization only if the following assumptions hold good:

i. There is no default risk of the bonds.

ii. The buy and hold strategy is adopted by the investor.

iii. There is no instantaneous change in interest rate during the investment horizon.

iv. The term structure is flat.

Mr. Suresh, a high income and high net worth investor is evaluating the following three alternatives for investing his surplus funds for medium to long-term period.

I. Secured, redeemable and non-convertible bonds are issued by Infrastructure Development Corporation as per the following terms:

Date of issue	Face value and maturity Value (Rs.)	Issue price (Rs.)	Annual Coupon rate	Maturity date
01.04.2005	1000	983	Prime Lending Rate + 0.5%	31.03.2010

The following interest rates materialize on the various reset dates:

Reset dates	31.03.2005	31.03.2006	31.03.2007	31.03.2008	31.03.2009
PLR (%)	10.25	10.05	9.80	9.60	9.50

Tax rate applicable is 30%.

II. IDBI Flexi bonds came out with an issue of discount bond. Each bond having a face value of Rs.6630 was issued at a discounted price of Rs.5000 with a maturity period of 5 years from the date of allotment. Assume a tax benefit of 15% on the invested amount.

You are **required** to:

a. Find out the coupon rate of bond issued by Infrastructure Development corporation assuming that Pure Expectations Hypothesis holds good.
b. Find out the post-tax YTM of the bond issued by Infrastructure Development Corporation.
c. Find out the post-tax YTM of the IDBI Flexi bonds.
d. Evaluate both the alternative investments for Mr. Suresh.

Answer

a.

Year	Expected annual PLR	Loading (%)	Coupon rate (%)
2005-06	10.25%	0.5%	10.75
2006-07	$[(1.1005)^2 \div 1.1025] - 1$ =9.85%	0.5%	10.35
2007-08	$[(1.098)^3 \div (1.1005)^2] - 1 =$ 9.3%	0.5%	9.8
2008-09	$[(1.096)^4 \div (1.098)^3] - 1 =$ 9%	0.5%	9.5
2009-10	$[(1.095)^5 \div (1.096)^4] - 1 =$ 9.1%	0.5%	9.6

b.

Year	Coupon	Tax on coupon @ 30%	Post-tax Cash flow
2005-06	107.5	32.25	75.25
2006-07	103.5	31.05	72.45
2007-08	98	29.40	68.60
2008-09	95	28.50	66.5
2009-10	96	28.80	67.2

Now, Post-tax YTM for Infrastructure Development Corporation:

$$983 = \frac{75.25}{(1+K)} + \frac{72.45}{(1+K)^2} + \frac{68.60}{(1+K)^3} + \frac{66.50}{(1+K)^4} + \frac{1067.20}{(1+K)^5}$$

Solving, by trial and error we get Post-tax YTM = 7.47%

c. Tax benefit on invested amount = 5000(15%) = Rs. 750

Maturity amount = 6630

Therefore, Actual investment amount = 5000 – 750 = 4250

$\therefore 4250(1+YTM)^5 = 6630$

$\therefore$ YTM = 9.3%

d. While investing in the bond, one should bear in mind that investing in a bond exposes to price risk and interest rate risk. The interest rate may change which will affect the YTM. The bond's price may quote at discount to face value which exposes one to price risk.

When we compare the YTM, IDBI Flexi bond scores higher. So in the above investment alternatives, IDBI Flexi bond is the better alternative.

Following are the yields on zero coupon bonds:

Maturity (Years)	YTM
1	10%
2	11%
3	12%

Assuming that the expectation hypothesis of term structure holds good, you are **required** to

a. Calculate the implied one-year forward rates and prices of the zero coupon bonds having a face value of Rs.1,000.

b. Calculate the expected yield to maturities and prices of one year and two year zero coupon bonds at the end of first year.

c. Calculate expected total return on the two bonds, if you have purchased two-year and three-year zero coupon bonds and held for a period of one year.

d. Calculate the current price of a 3-year bond having a face value of Rs.1,000 with a coupon rate of 11%. If you buy this bond at the current price and hold for one year, what is the expected holding period return?

Answer

2. a We can calculate forward rates by calculating the price of zero coupon bonds

Maturity	YTM	Price	Forward rate
1	10	$\frac{1,000}{1.10} = 909.09$	–

2	11	$\frac{1,000}{(1.11)^2} = 811.62$	$\frac{(1.11)^2}{1.10} - 1 = 12.01\%$
3	12	$\frac{1,000}{(1.12)^3} = 711.78$	$\frac{(1.12)^3}{(1.11)^2} - 1 = 14.03\%$

b. The next year's prices and yields can be calculated by discounting each zero's face value at the forward rates for the next year that we have calculated in part (a)

Maturity	Price	YTM
1 year	$\frac{1,000}{(1.1201)} = 892.78$	12.01%
2 year	$\frac{1,000}{(1.1201) \times (1.1403)} = 782.93$	13.02%

c. Next year, the 2 year zero coupon bond will be 1-year zero coupon bond and will therefore sell at $\frac{1,000}{1.1201} = 892.78$.

Similarly, the current 3-year zero coupon bond will be a 2-year zero and will sell at $\frac{1,000}{(1.1201) \times (1.1403)} = 782.93$.

Expected total return

2-year bond = $\frac{892.78}{811.62} - 1 = 9.99\% \cong 10\%$

3-year bond = – 1 = 9.99% ≅10%

d. The current price of the bond should equal the value of each payment times, the present value of Rs.1 to be received at the time of maturity. The present value can be calculated in the following manner:

= 109.09 + 97.39 + 797.19

= Rs.1,003.67

Similarly, the expected price after 1 year can be calculated using forward rates.

= 107.13 + 876.88 = 984.01

Total expected return

$$\left(\frac{120+984.01-1,003.67}{1,003.67}\right) = 9.99\% \cong 10\%$$

Q. India Plastics Ltd. have recently issued the floating rate bonds. The details of the issue is given under:

Date of issue	Face value and maturity value (Rs.)	Issue Price (Rs.)	Coupon Rate	Maturity Date
1.04.2004	1,000	950	Annual Prime Lending Rate + 0.65%. However minimum coupon is limited to 10.0%	31.03.2009

The term structure of Prime Lending Rate (PLR) for different maturity is given as under:

Maturity as on	31.03.2005	31.03.2006	31.03.2007	31.03.2008	31.03.2009
PLR (%)	9.00	9.15	9.55	10.00	10.15

Considering that the Pure Expectations Hypothesis holds good, you are **required** to

a. Calculate YTM for the bondholder who has recently been allotted India Plastic bonds at issue price.

b. Estimate duration of the Floating rate bond.

Answer

a.

Year	Expected Annual PLR	Loading (%)	Total (%)	Effective Coupon Rate (%)
2004-05	9.00%	0.65	9.65	10.00
2005-06	$[(1.0915)^2 \div 1.09] - 1 = 9.30\%$	0.65	9.95	10.00
2006-07	$[(1.0955)^3 \div (1.0925)^2] - 1 = 10.15\%$	0.65	10.80	10.80
2007-08	$[(1.10)^4 \div (1.0955)^3] - 1 = 11.36\%$	0.65	12.01	12.01
2008-09	$[(1.1015)^5 \div (1.10)^4] - 1 = 10.75\%$	0.65	11.40	11.40

Therefore, YTM denoted by **r** can be found out from following expressions

$$950 = \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{108}{(1+r)^3} + \frac{120.1}{(1+r)^4} + \frac{114+1000}{(1+r)^5}$$

Consider, r = 12%; R. H. S. = 954.3

At r = 13%; R. H. S. = 920.00

$$\text{i.e., } r = 12 + \frac{954.3-950}{954.3-920}$$

$$= 12.125 \cong 12.13\% \cong 12\%$$

b.

Year	C.F	PV. of C.F (12%)	PV. of C.F. x n
1	100.00	89.1822	89.1822
2	100.00	79.5346	159.0693
3	108.00	76.6052	229.8156
4	120.10	75.9724	303.8895
5	1114.00	628.458	3142.289
		949.752	3924.245

$$\text{Duration of Floating rate bond} = \frac{3924.245}{949.752} = 4.13 \text{ years}$$

Q. Mr. Jaypal works as a middle level manager in a Public Sector Undertaking. His gross total income is Rs.5,00,000 p.a. He wants to avail the benefit of tax rebate (@15 %) under section 88 of the Income Tax Act, by investing Rs.2,00,000 in the Tax Saving Bonds issued by the ICICI Bank. He approaches you for your advice. Options available to Mr. Jaypal in respect of Tax Saving Bonds are:

Option	Issue Price (Rs.)	Face Value (Rs.)	Tenure	Interest (%) (p.a.)	Interest Payable

I	10,000	10,000	4 years	5.65	Annually
II	10,000	10,000	6 years	7.00	Annually
III	10,000	14,750	4 years 9 months	DDB*	DDB*
IV	10,000	17,800	6 years 9 months	DDB*	DDB*

* Deep Discount Bond

The marginal tax rate applicable to Mr. Jaypal is 30 percent.

You are **required** to

a. Determine the post-tax YTM for the four options available to Mr. Jaypal. Assume that the interest income is tax exempt.

b. Suggest an option, if

i. The yield curve is upward sloping.

ii. The yield curve is downward sloping.

iii. The yield curve is flat.

Answer

OPTION – I

Intrinsic value or present value of the bond

= Coupon amount × PVIFA(i, n) + (Face value of the bond) PVIF (i, n)

where,

Coupon amount = FV × Interest rate

= 10,000 × 5.65%

= Rs.565

i = YTM

n = 4 yrs

∴ Net investment in the bond = Rs.10,000 × 0.85 (Since 15% tax rebate is available)

= Rs. 8500

Therefore

8500 = 565 × PVIFA (i,4) + 10,000 × PVIF (i,4)

at i = 10%

RHS = 8621.11

at i = 12%

RHS = 8071.30

by interpolation

$$\Rightarrow \quad 10\% + (2)\ \frac{8621.11-8500}{8621.11-8071.3}$$

⇒ 10 + (2) (0.0.2202)

= 10 + 0.4406

= 10.4406%

OPTION- II

Intrinsic value or present value of the bond

= Coupon amount × PVIFA(i, n) + (Face value of the bond) PVIF (i, n)

where,

Coupon amount = FV $\times$ Interest rate

= 10,000 $\times$ 7.00%

= Rs.700

i = YTM

n = 6 yrs

$\therefore$ Net investment in the bond = Rs.10,000 $\times$ 0.85 (Since 15 % deduction)

= Rs. 8500

$\therefore$ 8500 = 700 $\times$ PVIFA (i,6) + 10,000 PVIFA (i,6)

for a rate of i = 12%

RHS =Rs.7944.33

For a rate of i = 10%

RHS = Rs.8694.39

$\therefore$ By interpolation

$$10\% + 2\% \times \frac{8694.39 - 8500}{8694.39 - 7944.33}$$

10 + 2 $\times$ 0.2592 = 10.5184%

OPTION – III

Net investment in the bond = Coupon amount $\times$ PVIFA (i,n) + Face value $\times$ PVIF(i,n)

8500 = 0 $\times$ PVIFA (i,4.9yrs) + 13000 PVIFA (I, 4.75 yrs)

$$8500 = 14750 \times \frac{1}{(1+i)^{4.75\text{yrs}}}$$

$(1+i)^{4.75}$ = 14750/8500 = 1.7353

I = 12.3%

OPTION IV

According to the given values

$$8500 = \frac{17800}{(1+i)^{6.75}}$$

$(1+i)^{6.75}$ = 17800/8500 = 2.0941

I = 11.57%

b. i. When the yield curve is upward sloping, it indicates that the expected interest rates in the future are higher. Hence in such situations it is advisable to invest in short term bonds and reinvest the amount at a higher rate in future.

Hence the bond option I with least maturity of 4 years is suggested.

ii. When the yield curve is downward sloping ,it indicates that the expected interest rates in the future are lower. Hence in such situations it is advisable to invest in long term bonds. Because when the interest rates are expected to decrease it is advisable to lock the investment in long term investments. Hence the bond option IV with highest maturity 6 years 9 months is suggested.

iii When the yield curve is flat, it indicates that the interest rates are expected to remain at the same level. Hence in such a situation it is advisable to choose a bond option with highest yield to maturity. Hence the bond option III with highest YTM of 12.3% is to be selected.

Q. Mr. Reddy is holding two bonds A and B with an annual coupon of 6% and 8% and their terms to maturity are 4 years and 6 years, respectively. The face value and maturity value of the bonds is Rs.100. Spot rates prevailing in the market as indicated by the yield curve are:

Maturity (Years)	Spot rates
1	3.40%
2	3.55%
3	3.80%
4	4.20%
5	4.55%
6	4.80%

a. Using the duration concept, calculate the expected change in the prices of bonds A and B for a 0.50% change in yield to maturity.

b. Calculate the one year holding period return on the bonds assuming that spot rates will fall in twelve month's time by 0.25%, across the maturity spectrum.

Answer

$$P = \frac{6}{(1.0340)} + \frac{6}{(1.0355)^2} + \frac{6}{(1.0380)^3} + \frac{106}{(1.0420)^4}$$

P = 5.803 + 5.596+ 5.365+ 89.916

= Rs.106.68

YTM of the bond A

$$106.68 = \frac{6}{(1+k)^1} + \frac{6}{(1+k)^2} + \frac{6}{(1+k)^3} + \frac{106}{(1+k)^5}$$

At K = 4%

LHS = 107.26

Hence YTM is approximately 4%.

Duration of Bond A

Year	C.F	P.v. of C.F at 4%	Year x P.V. of C.F
1	6	5.77	5.77
2	6	5.55	11.10
3	6	5.33	15.99
4	106	90.61	362.44
		107.26	395.30

$$\text{Duration} = \frac{395.3}{107.26} = 3.685 \text{ years}$$

$$\text{Modified duration} = \frac{3.685}{1+.04}$$

= 3.54 years

For a 0.50% increase in YTM change in the price of the bond A

$$\frac{\Delta P}{P} = -3.54 \times 0.50$$

= – 1.77%.

Price of the bond A will decrease by 1.77%.

Price of the bond B

$$P = \frac{8}{(1.0340)^1} + \frac{8}{(1.0355)^2} + \frac{8}{(1.0380)^3} + \frac{8}{(1.0420)^4} + \frac{8}{(1.0455)^5} + \frac{108}{(1.0480)^6}$$

= 7.737 + 7.461+ 7.153+ 6.786+ 6.404+ 81.518= Rs.117.06

Yield to maturity of the bond B

$$117.06 = \frac{8}{(1+K)^1} + \frac{8}{(1+K)^2} + \frac{8}{(1+K)^3} + \frac{8}{(1+K)^4} + \frac{8}{(1+K)^5} + \frac{108}{(1+K)^6}$$

Or

117.06 = 8 x PVIFA (K_1 6) + 100 x PVIF (K_1 6)

at = K = 5%

L.H.S. = 115.21

at K = 4%

L.H.S = 120.94.

$$YTM = 4\% + \frac{(120.94 - 117.06)}{(120.94 - 115.21)}$$

= 4.68%.

Duration of the Bond B

Year	C.F	Present value of cash flow at (4.68%)	Year x PVCF
1	8	7.642	7.642
2	8	7.301	14.602
3	8	6.974	20.922
4	8	6.662	26.648
5	8	6.365	31.825
6	108	82.08	492.48
		117.02	594.119

$$\text{Duration} = \frac{594.119}{117.02}$$

= 5.077 years

$$\text{Modified duration} = \frac{5.077}{1+.0468}$$

= 4.850 years

Change in the price of the bond = –4.850 x 0.50

= – 2.425%

Therefore, price of the bond B will decline by 2.425%.

b. Price of the bonds after one year

Bond A

$$\frac{6}{(1.0315)^1}+\frac{6}{(1.0330)^2}+\frac{106}{(1.0355)^3}$$

= 4.85 + 4.69 + 94.70 = Rs.106.91

One year holding period return on bond A = $\frac{106.91-106.68+6}{106.68}$ = 5.84%

Bond B

$$\frac{8}{(1.0315)}+\frac{8}{(1.0330)^2}+\frac{8}{(1.0355)^3}+\frac{8}{(1.0395)^4}+\frac{108}{(1.0430)^5}$$

= 7.756 + 7.497+ 7.205+ 6.852 + 87.5= 116.81

One year holding period return = $\frac{116.81-117.06+8}{117.06}$ = 6.62%

Q.

Vishnu Ceramics Ltd is planning to issue floating rate bonds with 6-year maturity on 1.12.2006. The face value of the bonds is Rs.1,000 and the issue price is Rs.985. The coupon rate of the bonds is determined each year by loading 2.5% to the rate obtained from the following prices of zero-coupon bonds as of 1.12.2006:

Maturity date	Price of the bond (Rs.)
1.12.2007	943.40
1.12.2008	898.47
1.12.2009	847.62
1.12.2010	792.16
1.12.2011	754.35
1.12.2012	728.76

The coupon rate is restricted to a maximum of 9%, and it is assumed that the pure expectations hypothesis holds good.

You are **required** to

a. Compute the YTM of the given floating rate bonds.

b. Compute the price of the bonds for a 0.5% change in the YTM of the bonds in both directions using the duration concept.

c. Find out the effective duration of these bonds for a 0.5% change in the YTM of the bonds. What is your comment on the effective duration and modified duration?

(4 + 5 + 2 = 11 marks)

Answer

a. YTM of the bonds:

First we need to compute the coupon rates during the six years:

Maturity date	Price of the bond (Rs.)	Interest rate[1] (%)	Expected Interest rate (%)	Loading (%)	Total (%)	Coupon (%)
1.12.2007	943.40	6.00		2.5	8.50	8.50
1.12.2008	898.47	5.50	$1.055^2/1.06 - 1 = 5.00$	2.5	7.50	7.50
1.12.2009	847.62	5.67	$1.0567^3/1.055^2-1 = 6.00$	2.5	8.50	8.50
1.12.2010	792.16	6.00	$1.06^4/1.0567^3-1 = 7.00$	2.5	9.50	9.00

1.12.2011	754.35	5.80	$1.058^5/1.06^4-1 = 5.00$	2.5	7.50	7.50
1.12.2012	728.76	5.41	$1.0541^6/1.058^5-1 = 3.48$	2.5	5.98	5.98

1 For a zero coupon bond, Price = Face value/$(1+r)^n$

YTM of the bonds:

$$985 = \frac{85}{(1+r)^1} + \frac{75}{(1+r)^2} + \frac{85}{(1+r)^3} + \frac{90}{(1+r)^4} + \frac{75}{(1+r)^5} + \frac{1,059.8}{(1+r)^6}$$

'r' is the YTM in the equation and it can be found out by trial and error.

For r = 8%, R.H.S = Rs.995.53

For r = 9%, R.H.S = Rs.951.17

$$\text{Therefore YTM} = 8\% + \frac{995.53 - 985.00}{995.53 - 951.17} = 8\% + 0.24\% = 8.24\%$$

b. In order to compute change in the price of the bonds for a specified change in YTM, we need to first compute the duration of these bonds:

Year (1)	Cash flow (2)	PV of cash flow @ 8.24% (3)	Col (2) x Col (3)
1	85	78.53	78.53
2	75	64.02	128.04
3	85	67.03	201.09
4	90	65.57	262.28
5	75	50.48	252.40
6	1059.8	659.02	3,954.12
		984.65	4,876.46

$$\text{Duration} = \frac{4,876.46}{984.65} = 4.95 \text{ years}$$

$$\text{Modified duration} = \frac{D}{1+y} = \frac{4.95}{1+0.0824} = 4.57 \text{ years}$$

Now, for 0.5% increase in YTM, change in price of the bonds:

$$\frac{\Delta P}{P} = -D_{mod} \times \Delta y$$

$= -4.57 \times 0.5 = -2.285\%$

Price of the bonds = 985 (1 – 0.02285) = Rs.962.49

For a 0.5% decrease in YTM, change in price of the bonds:

$$\frac{\Delta P}{P} = -D_{mod} \times \Delta y$$

$= -4.57 \times -0.5 = 2.285\%$

Price of the bonds = 985 (1 + 0.02285) = Rs.1,007.5

Effective duration:

$$D_{eff} = \frac{P_2 - P_1}{2P_0 (\Delta y)}$$

where,

P_2 = the estimated price of the asset after a downward shift in the interest rates

P_1 = the estimated price of the asset after an upward shift in the interest rates

P_0 = current price of the bonds

Δy = change in the yield

$$D_{eff} = \frac{1,007.5 - 962.49}{2 \times 985 \times 0.005} = 4.57 \text{ years}$$

Comment: Since there is no call option embedded in the bonds, the effective duration is same as modified duration.

Q.

Mr. Manish, an analyst, is very much concerned about the continuous movement of the interest rates. One of his clients, Mr. Amish would like to put a fixed part of his investible funds in fixed income securities. Hence, Mr. Manish is currently analyzing two recently issued bonds, the details of which are as follows:

Particulars	Bond A	Bond B
Maturity	5 years	5 years
Coupon	9.50% p.a.	10% p.a.
Feature	Non-Callable	Callable
Call date	—	3 years from the date of issue
Issue price	Rs.950	Rs.1000
Face value	Rs.1000	Rs.1000
Periodicity of interest payment	Annual	Semiannual
Call price	—	Rs.1100

You are **required** to calculate

a. The effect on the price of Bond A for a 50 basis points decrease in the market interest rates.

b. Duration to call of bond B.

(6 + 4 = 10 marks)

Answer

a.

YTM for Bond

950 = 95 × PVIFA(K%,5) + 1000 × PVIF(K%, 5)

At the rate of 10% R.H.S = 981.03

At the rate of 12% R.H.S = 909.86

By doing interpolation YTM = 10.87% p.a.

Duration of Bond A

Period	CF	PVIF@10.87%	PV(CF)	n × PV(CF)
1	95	0.902	85.69	85.69
2	95	0.813	77.24	154.48
3	95	0.734	69.73	209.19
4	95	0.662	62.89	251.56
5	1095	0.597	653.72	3268.6
			949.27	3969.52

Duration = $\frac{3969.52}{949.27}$ = 4.18 years.

MD of Bond A

MD = $\frac{D}{1+0.1087}$ = $\frac{4.18}{1.1087}$ =3.77

% Price Change = –3.77 × $\frac{-50}{100}$ = 1.885% (increase)

b. Yield to call for Bond B

1000 = 50× PVIFA(K%,6) + 1100 × PVIF(K%, 6)

At the rate of 6% R.H.S. = 1021.37

7% R.H.S. = 971.26

By, doing interpolation, YTM = 6.43% half yearly

Duration to call for Bond B

Period	CF	PV@6.43%	PVCF	PV × n
1	50	0.940	47	47
2	50	0.883	44.15	88.30
3	50	0.829	41.45	124.35
4	50	0.779	38.95	155.80
5	50	0.732	36.60	183.00
6	1150	0.688	791.20	4747.20
			999.35	5345.65

Duration to call = $\frac{5345.65}{999.35}$ = 5.35 half yearly period. i.e 2.67 years.

Theory Questions

Q. Briefly explain the various types of commonly used duration measures.

Some of the durations, which are commonly used, are as under

Macaulay Duration

This duration is calculated as the "present-value-weighted time to receipt of cash flows," which is why it is quoted in "years." Each cash flow "time" is multiplied by the present value of the associated cash flows and then the sum of all of these terms is divided by the sum of the present values.

Modified Duration

It presented a proof of the relationship between duration and bond price changes. Modified duration is a measurement of the change in value of an instrument in response to a change in interest basis (payment frequency). This "modifies" Macaulay duration. The relationship of duration and price volatility can be expressed as follows:

Percentage price change = -Modified duration x Yield change x 100

Key Rate Durations (Partial Duration)

Measures the price sensitivity of a bond (or a portfolio of bonds) to changes in specific parts of the yield curve.

Spread Duration

A measure of the percentage price change to a change in the spread (OAS) of a bond. This is a very important duration measurement for floaters.

Empirical Duration

Empirical duration was developed to deal with how the security has been trading instead of *estimating* how a security will trade. In other words, it uses historical measurements that calculate actual price changes and changes in the level of the market to measure how the security is actually performing.

Constant Dollar Duration

Constant dollar duration measures duration for securities in particular price ranges and is mainly used in mortgage-backed securities.

Portfolio Duration

The price sensitivity of an entire portfolio as an aggregated unit, versus the weighted average price sensitivity of each individual security.

Q. 'Both Modified Duration and Effective Duration are the ratio of the proportional change in bond value to the parallel shift of the spot yield curve.' So, is it correct to assume that both these measures are calculated the same way and both can be used interchangeably for estimating the price change? Discuss.

Answer

Both Modified Duration and Effective Duration are the ratio of the proportional change in bond value to the parallel shift of the spot yield curve. They both estimates bond price changes.

Duration estimates bond price changes by the use of the following formula

Percentage price change = (Modified duration) (Change in yield)

Modified duration = $\frac{Duration}{1+Y/f}$

Where, y = YTM in decimal form and f = frequency of discounting

The formula for calculating effective duration is,

Effective duration = $\frac{P_2 - P_1}{2P_0(\Delta y)}$

P_2= The estimated price of the asset after a downward shift in the interest rates

P_1= The estimated price of the asset after an upward shift in interest rates

P_0= The current price of the asset

Δy = The assume shift in the yield

The difference between the two is that modified duration can only be used for fixed-rate bullet securities. The derivative of bond price volatility relative to yield changes that result in the "modified duration" formula has embedded in it the assumption that the cash flows of the bond do not change as rates change. This means that modified duration formula is inappropriate for any bond where the cash flows change as interest rates change. Effective duration takes into account these changes in cash flows of the bond. Effective duration makes it possible to arrive at negative duration or durations longer than maturity of the asset both, which are not possible with modified duration. It also takes into account the changes in the value of bond due to the embedded option in the bond. Effective duration requires the use of an interest rate model and corresponding price model that will help in determining the prices for the asset when interest rates and cash flows change.

Q. According to the caselet, Bond ETFs differs from Bond Ladders. Discuss. Also state the disadvantages attached with Bond ETFs. Bond ETFs are given priority over Index Funds. Discuss

The liquidity and transparency of an ETF offers advantages over a passively held bond ladder. Bond ETFs offer instant diversification and a constant duration, which means an investor needs to make only one trade to get a fixed-income portfolio up and running. A bond ladder, which requires buying individual bonds, does not offer this luxury. One disadvantage of bond ETFs is that they charge an ongoing management fee. While lower spreads on trading bond ETFs help offset this somewhat, the issue will still prevail with a buy-and-hold strategy over the longer term. The initial trading spread advantage of bond ETFs is eroded over time by the annual management fee.The second disadvantage is that there is no flexibility to create something unique for a portfolio. For example, if an investor is looking for a high degree of income or no immediate income at all, bond ETFs may not be the product for him or her.

Bond ETFs and index bond funds cover similar indices, use similar optimization strategies and have similar performance. Bond ETFs, however, are the better alternative for those looking for more flexible trading and better transparency. The composition of the underlying portfolio for a bond ETF is available daily online, but this type of information for index bond funds is available only on a semi-annual basis. Furthermore, on top of being able to trade bond ETFs throughout the day, active traders can enjoy the ability to use margin, sell short and trade options on these securities.

Q. Explain the effect on the price of a call option and a callable bond compared to a non callable bond, with respect to changes in the interest rate.

Answer

The sensitivity of the price of the callable bond to the change in the interest rate depends on the changes in the components of the callable bond to the changes in the interest rates. The price volatility of a non-callable bond to the changes in the interest rates depends on its duration. If interest rate rise, the price of the non-callable bond will fall and the price of the call option also will fall. The decline in the price of the call option is because it becomes less valuable as interest rate rise. The decline in the call price will reduce the price of the callable bond too but the impact of the interest rate rise on the price of the non-callable bond will be greater than that on a callable bond. When the interest rates fall, both the price of the non-callable bond and the price of the call option increase. This will cause the price of a callable bond also to increase but to a lesser extent.

Q. How is option adjusted duration defined? Explain the important factors influencing option adjusted duration.

Answer

Option Adjusted Duration is the modified duration of a bond after adjusting for any embedded optionality. The Option Adjusted measure of duration takes into account the fact that yield changes may change the expected cash flows of the bond because of the presence of an embedded option, such as a call or put. Option-adjusted Duration is defined as follows:

Option-adjusted Duration = $\frac{Price_{NCB}}{Price_{CB}} \times Dur_{NCB}(1 - Delta)$

Where,

$Price_{NCB}$ =Price of a non-callable bond

$Price_{CB}$ =Price of a callable bond

Dur_{NCB} =Duration of the non-callable bond

Delta =Delta of the call option

Note that the delta of a call option measures the sensitivity of the option price to the changes in the price of the underlying asset. The delta value of a call option lies between 0 and 1.

Important factors influencing option-adjusted duration are:

a) The ratio of the price of the non-callable bond to the price of the callable bond. But these two prices are influenced by the price of the call option. Hence the higher the price of the call option, the higher the value of the ratio. Thus Option-adjusted Duration is indirectly dependent on the price of the call option.

b) The duration of the corresponding non-callable bond.

c) The delta of the call option.

The Option-adjusted Duration of a deep discount callable bond is the same as the duration of a non-callable bond. The Option-adjusted Duration of a premium callable bond in which the coupon rate is significantly higher than the current market yield is zero.

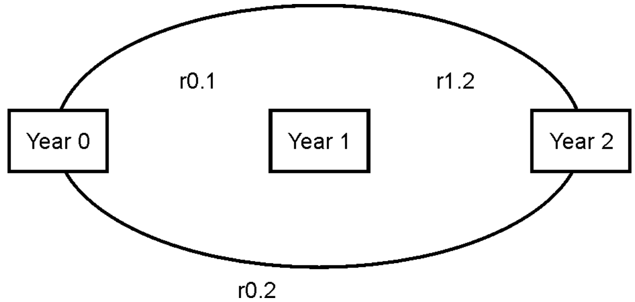
Q. Low interest rates and a flat yield curve are both vital to the growth of domestic economy. While the shape of any yield curve is a function of numerous fiscal and monetary factors, perceptions play a very important role in shaping the empirical yield curves. Using pure expectations theory, explain the upward sloping, inverted and flat yield curves.

Answer

Low interest rates and a flat yield curve both are vital to the growth of domestic economy. While the shape of any yield curve is a function of numerous fiscal and monetary factors. Perceptions play a very important role in shaping the empirical yield curves.

Pure Expectations Theory tries to explain the phenomena regarding the existence of different shapes of yield curves.

According to Pure Expectations Theory, the current term structure of interest rates are determined by the consensus forecast of future interest rates. This can be understood by considering the following hypothesis through which we can understand how the perceptions of investors regarding the interest rates shape the yield curve.



At time 0 there is short term interest rate r0, 1 for money borrowed in year 0 and repayable in year 1.There is also a long term interest rate r0,2 for money borrowed in year 0 and repayment in year 2. Linking these two rates is an unobservable "forward " that is expected to prevail in year 1 for money to be borrowed then for repayment in year 2 .In terms of this forward rate, one can write the arbitrage condition as

$(1 + r0,2)^2 = (1+r0,1)\ (1+r1,2)$

This says the total money (principal plus interest) repaid in year 2 should be the same whether the money is borrowed long-term at r0,2 or borrowed short-term at r0,1 and then "rolled-over" in year 1 at the then prevailing short-term rate r1,2 .The same condition holds for the investor also. The arbitrage condition says that the investor must be indifferent between these two alternatives.

Here we try to explain the shaping of yield curve with respect to the above theory by considering the following example of three different situations.

If one year interest rate is 15% (r0,1 = 15%) but

(i) is expected to go up to20%(r1,2=20%) at the end of one year

(ii) is expected to fall down to (r1,2 = 10%)

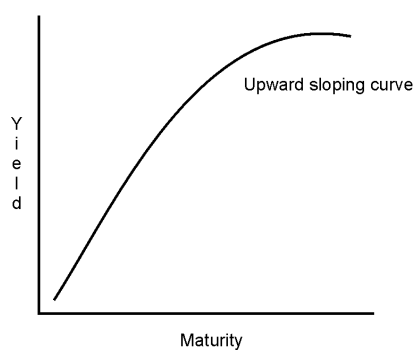
(iii) is expected to be the same.

Hence considering the first situation

(i) $(1+r0,2)^2 = (1+0.15)(1+0.20) \rightarrow (r0,2)= 17.5\%$

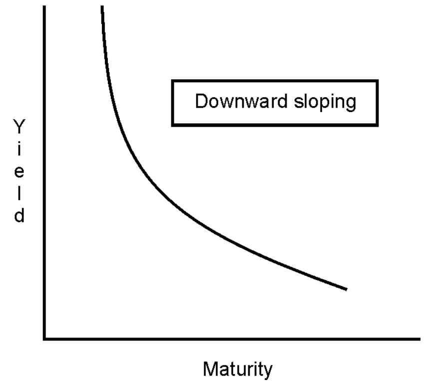
That is, an investor will opt for one year security now only when he is certain that the interest rate after one year is greater than the interest rate on two year security.

An upward sloping Yield curve according to this theory indicates that the investors expects that the interest rates going to rise.

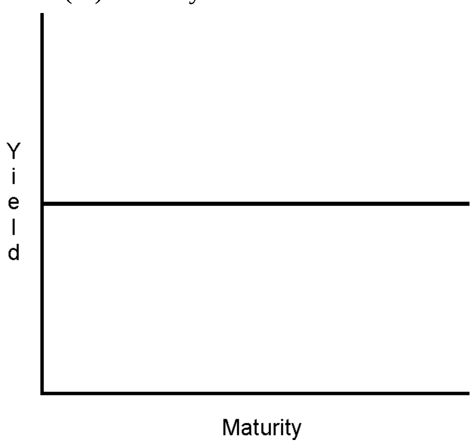


(ii) $(1+r0,2)^2 = (1+0.15)(1+0.10) \rightarrow (r0,2)= 12.5\%$

When interest rate on one year security is going to decline after one year, he will opt for two year instrument. A downward sloping curve according to this theory indicates that the investors expect a fall in interest rates.



(iii) A flat yield curve indicates that investors expect that the interest rates remain at the same level.



Q. The caselet mentions that there have been ‘good’ reasons for prices of riskier corporate bonds to climb compared with those of Treasuries. What according to you are the reasons for this anomaly?

Answer

The reasons why the prices of riskier corporate bonds have increased compared with those of Treasuries are:

i. Increasing profits have gone some way to improve the balance sheets of these companies. The proportion of companies with a “speculative grade” (junk) rating by Standard & Poor's, a big rating agency, that are defaulting has fallen from a peak of 10.5% in March 2002 to 2.3% last month, the lowest since 1998. And although S&P is still downgrading more companies than it is upgrading, the gap is shrinking.

ii. Increased risk appetite of investors. A hunger for anything with a sniff of yield attracts the investors and they do not worry much about the underlying risks.

iii. It is a self-feeding mechanism – investors are buying more of junk bonds than the government Treasuries and hence the increasing demand for junk bonds is fuelling the prices of junk bonds.

iv. This is also due to huge amounts of funds flowing to junk funds – mutual funds which specialize in investing in junk bonds. Since the high-yield market is fairly illiquid, similar to that for small-cap stocks, steep inflows to junk funds can boost prices as managers buy up shares with the new cash. However, big gains by junk bonds in general, and battered telecom junk specifically, might have as much to do with return-chasing as the merits. That means that bond prices could start to sink when flows to high-yield funds slacken.

v. Due to the Iraq war, risk-averse investors had little interest in loaning money to struggling companies. The yield spread between the average junk bond and the ten-year Treasury yawned to about 11 percentage points -- more than double its average -- as junk bond prices fell. With such high yields, junk was a bet that was hard to pass up.

Q. Caselet mentions that spreads are a poor guide to value, especially when yields are low. Comment.

This is because investors should be looking not at relative value, but for the same high absolute returns that they would require of similarly risky equities. Junk, or high-yield bonds, is debt issued by companies with a low credit rating due to short or spotty records of accomplishment. Since companies with a lot of debt offer them, less than perfect credit, or both, they tend to pay far higher interest than investment-grade bonds. It sounds good, but investors only get that interest if the issuer does not default. Now when the investors look at the yield spreads of junk bonds and Treasuries, the spread is definitely attractive. However, it is not an indicator of value because the two belong to different risk class. Treasury bonds are highly safe with almost negligible risk, whereas junk bonds are highly risky. That is why the spread may alone not be the true indicator of value.